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**Human Capital and Competition:
Strategic Complementarities in Firm-based Training**

SKOPE Research Paper No. 114 January 2013

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ISSN 1466-1535

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Abstract

Vocational training systems differ markedly between countries. A model of firm- based human capital investment predicts equilibria characterised by particular patterns of training and job-to-job mobility, consistent with observed cross-country differences. Incentives to invest in human capital are determined jointly with labour turnover and the intensity of competition between employers for skilled workers, and the dependence of labour market conditions on human capital leads to strategic complementarity between training decisions. Depending on the extent of market frictions and match heterogeneity, we may expect to see either equilibria characterised by general training, steep wage profiles and high mobility; or equilibria in which both general and specific investment may occur, but turnover is low and wage profiles are relatively flat. Multiple equilibria are possible, in which case high turnover equilibria generate higher welfare.

JEL Classification Codes: J24, J63

Keywords: Human capital, labour turnover, specific training, general training, search, matching.

1 Introduction

Why do vocational training systems differ so much between countries? Distinctive examples such as the German apprenticeship system and the Japanese “lifetime employment” tradition give rise to very different patterns of on-the-job training from each other, and from the more diffuse arrangements in the US and the UK. Descriptive analyses have categorised the German pattern as a *high-skills equilibrium*, in contrast to the *low-skills equilibria* in the US or UK. This paper provides an explanation for the diversity of training institutions and outcomes.

I present a model of training investment in a frictional labour market with endogenous job creation. I show that strategic complementarities in on-the-job training decisions can explain the contrasting training and labour market equilibria that we observe. In this environment, when other firms and workers are investing in specific training, the labour market becomes more favorable to specific investment: since specifically-trained workers are less likely to change jobs, expected turnover in the labour market is low, and the returns to specific training are high, while limited future employment opportunities lower the returns to general training. Conversely, when others invest in general training turnover is high – raising the incentive to create jobs for skilled workers and hence boosting the returns to general training, while low expected tenure discourages specific investment.

Depending on the degree of friction, the technology of training, and the potential gains from labour turnover, we may see either equilibria with high turnover, general training financed mainly by workers, and no specific training; or low turnover equilibria reinforced by high investment in specific training, in which costs of both general and specific investment are shared between workers and firms and the gains from turnover are foregone. We also have the possibility of multiple equilibria, in which case the high-turnover general-skills equilibrium is superior.

As we might expect, general training is associated with a more competitive labour market equilibrium, and specific training with a labour market in which the intensity of competition between firms is low. That the degree of labour market competition affects training incentives is well-understood: for example it has been shown (Stevens, 1994; Acemoglu, 1997; Acemoglu and Pischke, 1999) that an imperfectly competitive labour market can explain why firms bear at least some of the costs of general training, in apparent contravention of Becker’s famous (1962) result. But in this paper causality also runs in the opposite direction: the labour market environment determines, and is determined by, the training decisions of individual agents.

Labour turnover is the key to the results. When it takes time to find a partner in

the labour market, the vacancy-filling and job-finding rates signal the supply and demand for skill to market participants. Turnover is modelled in a general equilibrium search and matching framework, with match heterogeneity and on-the-job search. Frictions and match heterogeneity are well-documented features of labour markets, and returns to worker mobility, as well as human capital, account for a substantial part of wage growth (Topel and Ward, 1992; Manning, 2003; Dustmann and Pereira, 2008). As Pischke (2007) points out, labour turnover is often assumed to be bad for investment in skills, but job search is itself a form of human capital investment. And it is the potential for turnover, and uncertainty about how long a match will last, that is at the heart of most interesting questions in the economic analysis of investment in training.

The only market imperfection in the model is the friction that means firms and workers are not instantaneously aware of potential matches – the meeting rate is determined by a matching function. I abstract from a variety of other imperfections that may be important in practice, such as credit constraints causing underinvestment in general skills by employees; and asymmetries of information (Katz and Ziderman, 1990; Chang and Wang, 1995; Acemoglu and Pischke, 1998; Autor, 2001), minimum wages, and unions, all of which may compress the wage structure and enhance training incentives for firms (Acemoglu and Pischke, 1999) while reducing the return to employees. I assume that training is fully contractible (there is no hold-up problem), and that wages are determined by bargaining under full information so that labour turnover is privately efficient. Thus the existence of different types of equilibria – high-turnover general-training equilibria, and low-turnover specific-training equilibria – is a generic property of human capital investment, rather than a result of contracting or information problems.

1.1 Human Capital Theory and Cross-Country Evidence

The characteristics of training systems can be interpreted in terms of the basic theory of general and specific training first set out by Becker (1962). If training is general (of equal value to many employers) and the labour market is competitive, the worker captures the return to an investment in on-the-job training in the form of higher future wages, and hence (in the absence of credit constraints) it is financed by the worker rather than the employer. The polar case is specific training, valuable only in a single firm, which has no effect on the competitive wage. If there are no contracting problems the wage profile of a specifically trained worker is indeterminate, but Becker suggested that the returns, and also the costs, would be shared between worker and firm.

An extensive literature has explored and extended Becker's original contribution. It has been shown (Stevens, 1994; Acemoglu, 1997; Acemoglu and Pischke, 1999) that

an imperfectly competitive labour market can explain why in practice firms seem to bear at least some of the costs of general training. For specific training, a rising wage profile consistent with Becker's hypothesis may result from long-term wage contracts in response to either ex-post information asymmetries (Hashimoto, 1981), or a hold-up problem (Macleod and Malcomson, 1993) which can also induce firms to invest in general skills (Kessler and Lülfesmann, 2006). With imperfect competition due to match heterogeneity and a limited number of firms, specific training can raise the wage when there is no long term contract, since it affects the wage offers of competing firms (Scoones, 2000; Stevens, 2001).

A useful international comparison was provided by Lynch (1994), who summarised the characteristics of training systems of ten OECD countries including the US, UK, Germany, Japan and France; the stylised facts remain broadly true today. Japan is characterised by employment patterns involving low labour mobility and long tenure, enabling employers to finance investment in skills that are technologically general: high turnover costs for workers and the consequent lack of effective labour market competition allow them to recoup training costs by paying below the competitive wage. However, long tenure also ensures that the return to specific investment is high. Maki, Yotsuka and Yagi (2005) claim that in the Japanese system workers mainly developed specific skills, which impeded the reconstruction of the economy following the stagnation of the 1990s.

Lynch (1994) described the German pattern as a high-skill, high-productivity equilibrium. In the German "dual system", almost two-thirds of school leavers enter apprenticeships providing in-firm vocational training. The curriculum is carefully regulated, and in contrast to Japan, apprenticeships provide German workers with highly marketable general skills, transferable across a wide range of occupations (Clark and Fahr, 2002). The apparent willingness of German firms to provide general training has sometimes been regarded as a puzzle, but Oulton and Steedman (1994) argued that it was financed mainly by workers accepting low trainee wages. Mohrenweiser and Zwick (2009) argue similarly that the German system can be reconciled with theory: they show that in commercial and trade occupations for which skills are indeed general and skilled workers are mobile, apprentice wages are sufficiently low that apprenticeships are profitable for firms, while in manufacturing occupations firms incur net training costs, but skills are more specific and the post-apprenticeship retention rate is high.

It is more difficult to provide a simple characterisation of US training. Lynch described the US system as highly decentralised, and firm-based training as mainly specific. The lack of nationally recognised vocational qualifications inhibits employee investment in general training. This is consistent with the comparative results in Leuven and Ooster-

beck (1999) from the International Adult Literacy Survey. They find that the volume of work-related training per worker is low relative to Canada, Netherlands and Switzerland, and the proportion financed by the worker is also lower in the US¹. However training is mainly employer-financed even when provided by an external organisation, which the authors interpret as evidence of firms paying for general training in an imperfectly competitive labour market. Loewenstein and Spletzer (1998) and Barron, Berger and Black (1999) present further evidence that on-the-job training in the US is partially general but mainly financed by employers². Low investment by workers might be explained by unenforceability of firm-based training contracts (Malcomson et al, 2003). Dustmann and Schönberg (2012) attribute the relative lack of success of apprenticeship in Anglo-Saxon countries to firms' inability to commit to training provision in the absence of the institutional support available in Germany, Austria and Switzerland. They argue that low apprentice wages and dropout rates in Germany relative to the UK are consistent with differences in commitment.

Empirical evidence on the relationship between training and turnover is hard to interpret, particularly because it is rarely possible to distinguish between general and specific training. At the individual level, there is some evidence of a small negative (Lynch, 1991; Parent, 1999; Garloff and Kuckulenz, 2005) or negative but insignificant (Krueger and Rouse, 1998; Bassanini et al, 2007) effect of training on the probability of quitting. Pischke (2007) discusses the difficulty of using data on turnover, tenure, or wages to distinguish between competitive and imperfectly competitive models of training without independent information on the type of training.

1.2 Multiple Equilibria

The possibility of multiple skills equilibria has been widely discussed. Finegold and Soskice (1988) suggested that Britain was trapped in a “low-skills equilibrium . . . a self-reinforcing network of societal and state institutions which interact to stifle the demand for improvements in skill levels”, while Acemoglu and Pischke (1998) characterise the German training system as a “high-training low-quits” equilibrium, in contrast to the “low-training high-quits” equilibrium in the US. In their model workers are credit constrained, but on-the-job general training is financed by firms, who have ex-post monopsony power due to

¹As reported by workers, 10% of training is financed by themselves, 85% by the firm, and 6% by government.

²For the UK, IALS data presents a similar picture to that for the US (Leuven, 2001); see also Bassanini *et al* (2007). The proportion financed by workers is even smaller (2%), despite the existence of recognised vocational qualifications.

adverse selection: high ability workers do not quit because their ability is not observed by competing firms. There may in principle be more than one solution to the equilibrium conditions; if so, an equilibrium with lower quitting has correspondingly higher training.

A number of theoretical models have demonstrated multiple equilibria arising from a coordination problem between workers and firms when there are labour market frictions and investments must be made before entering the market. Redding (1996) and Acemoglu (1996) have equal numbers of workers and firms, investing in human and physical capital (or R&D in Redding’s growth model) before discovering the identity of their labour market partner. In the models of Laing et al (1995) and Burdett and Smith (2002), workers make ex-ante investments in education and job creation is endogenous. None of these models allows contracting between firms and workers on educational investment, and all human capital is general.

The question of stability has often been ignored. In Redding (1996), Acemoglu (1996) and one of the models in Burdett and Smith (2002), choices are dichotomous and there are two stable pure strategy equilibria (either all workers or no workers are trained). But in other cases, with continuous training outcomes, some of the equilibria that have been identified are unstable. In Snower (1996), and in the continuous model of Burdett and Smith (2002), the low skill equilibrium is unstable, as is the intermediate equilibrium of Laing et al (1995). Acemoglu and Pischke (1998) illustrate multiple equilibria in their model with an example in their (1996) working paper: in this case the “high-training low-quits” equilibrium is unstable (as pointed out by Dustmann and Schönberg, 2010).³

In contrast, the model in this paper demonstrates the possibility of multiple stable training equilibria when there are no contracting problems, asymmetries of information, or credit constraints. It also differs from other multiple equilibria models in allowing for both specific and general investments.

1.3 Outline of the Paper

In the next section I describe the search and matching environment, and the wage determination process. Then in section 3 the choice of human capital investments is determined, taking the job-finding rate for skilled workers as given. In section 4 I solve for the steady-state equilibria in the unskilled and skilled labour markets. The skilled job-finding rate, and hence the equilibrium intensity of competition, is determined by the job-creation decisions of firms. The character of the equilibrium and the strategic relationships between

³Each of these models has a zero-profit condition, and the possibility that profit might rise rather than fall in response to a perturbation has not been considered.

human capital investment and job creation are discussed, and the possibility of multiple equilibria is demonstrated. Implications are discussed in the last two sections.

2 The Labour Market Environment

Workers begin their lives without skills, and search for jobs in the unskilled labour market. Training decisions are made at the beginning of an employment spell. Once trained, workers may search in the skilled labour market: both employed workers, and those who are unemployed following a job destruction shock, search for skilled jobs. Firms may create skilled or unskilled vacancies, and in each market the rate at which firms and workers meet is determined by a matching function.

Agents are homogeneous, risk-neutral, and do not discount the future. The population of workers has unit mass; workers die at rate γ and are replaced by new workers, who enter the unskilled labour market and meet potential employers at rate λ_0 . Human capital investments take place instantaneously when a worker meets a firm and decide to match. The cost of any unit of human capital, whether general or specific, is normalised to one.

A worker with general and specific human capital (g, s) generates a flow of productivity $p_g(g) + p_s(s)$ in his current match. p_g and p_s are increasing, and concave at high levels of human capital, and $p_s(0) = 0$. The additive specification implies that there is no technological interaction between general and specific capital: they can be chosen independently. Matches are destroyed at rate δ , after which the worker enters skilled unemployment with human capital $(g, 0)$. Unemployed workers, whether skilled or unskilled, receive a flow of utility $b > 0$, which is lower than their productivity while employed: $b < p_g(0)$.

Skilled workers, whether employed or unemployed, search in the skilled labour market, and meet firms at rate λ .⁴ If a worker with human capital (g, s) moves to a new match his human capital, unless he invests further, will be $(g, 0)$.

Turnover occurs because matches are also characterised by ex-post heterogeneity. Each new match generates an idiosyncratic benefit z (or cost, if negative), realised when the worker and firm meet. z is a random variable with log-concave density, distribution function $F(z)$ and support $[-\infty, \bar{z}]$. When deciding whether to consummate a match, they will take this benefit into account, in addition to the productivity of human capital and the costs of new human capital investment.⁵ For simplicity, z is assumed to be an instan-

⁴The rate at which a worker meets firms in the skilled labour market is assumed to be independent of g . This rules out equilibria in which ex-ante identical workers make different choices of g in anticipation of entering different labour markets.

⁵Since we will assume below that the match surplus is always shared by full-information bargaining,

taneous benefit, received on consummation of the match at the same time as the costs of training are incurred. Using this specification – rather than, say, an idiosyncratic component of match productivity – is convenient for two reasons: it simplifies the analysis because the idiosyncratic element does not affect match duration, and it clearly differentiates the effects of the gains from turnover from those of specific capital.⁶ Likewise the effects of general and specific capital are separated by the additive specification for productivity.

Each firm has a single job which may be filled or vacant at any instant. Firms may create vacancies in either the skilled labour market or the unskilled labour market; their decisions determine the job-finding rates λ and λ_0 . The flow cost of a vacancy in either market is c , and free entry ensures that the expected present value of a vacant job is zero in both markets. Note that (as in other search models) the cost parameter c can be thought of as capturing the degree of exogenous friction in the market – it is the vacancy cost relative to the filling rate that matters for job creation. As c approaches zero, rapid job creation ensures that workers find jobs immediately and unemployment disappears.

2.1 Bargaining

I will assume throughout that the costs and benefits of matching are shared between workers and firms as a result of wage bargaining under full information, and consequently that all turnover decisions are privately efficient.

Cahuc, Postel-Vinay and Robin (2006) present a model of on-the-job search and strategic wage bargaining. When an unemployed worker meets a firm they bargain over the wage, which can then be renegotiated only by mutual agreement. Hence the wage remains constant until the worker encounters an alternative employer, at which point a three-way bargaining game determines the subsequent wage and employment – which prevail until a further opportunity occurs. In the three-way game, the two firms first make simultaneous wage offers; the worker can preliminarily accept one and then bargain in a standard alternating offer game with the other firm. This model delivers the generalised Nash Bargaining solution, in which the firm and worker obtain shares β and $1 - \beta$ of the match surplus (which is the difference between that match value and unemployment when the worker is recruited for unemployment, and subsequently the difference between the two match values). Bargaining power $\beta \in (0, 1)$ is determined by the relative time delays

it does not matter whether z accrues initially to the worker or the firm.

⁶An alternative specification, in which matches begin with an idiosyncratic stock of specific human capital which can be supplemented by investment, is tractable and delivers similar results, but is considerably more cumbersome.

between alternating offers.⁷

Note that for the Nash solution to emerge from the strategic bargaining game requires that the outside option can be used as a threat point. This seems reasonable for an unemployed worker, who can continue to meet other searching firms during wage negotiation; similarly an employed worker can continue in employment while bargaining with another firm. But it is less plausible to suppose that a worker can move temporarily to an alternative firm while negotiating with the original employer. Even if, as Cahuc *et al* argue, it is acceptable in their context, it is implausible when matches involve initial specific investments (Macleod and Malcomson, 1993) so I cannot adopt their model in full. Instead I modify it by assuming that if the worker accepts the initial outside wage offer he is unable to negotiate further with the original employer. This leads to the following outcome, in which turnover is still efficient:

1. A worker recruited from unemployment receives a wage which is the outcome of a Nash bargain with continued unemployment as the worker's threat point.
2. When the worker encounters an alternative potential employer:
 - (a) If the joint value of the potential alternative match is less than the value to the worker of his current contract, nothing happens.
 - (b) If the joint value of the alternative match is higher than the value to the worker of the current contract but lower than the joint value of the current match, the worker remains with the current employer but the wage is raised to give him a contract of the same value as the alternative match.
 - (c) If the alternative match is of value higher than the joint value of the current match the worker changes jobs, with a wage which is the outcome of a Nash bargain in which the worker's threat point is continued employment in the original firm in a contract giving him the full value of the match.

The difference from Cahuc *et al* is at point 2b: when remaining with his current employer the worker cannot push his wage *above* what he could obtain elsewhere. In addition bargains must allow for the instantaneous benefit z , and training costs, which occur at the start of the match. I assume that neither party is credit-constrained, so they may bear appropriate shares of training costs, and they include these initial costs and benefits in the determination of the initial wage.⁸

⁷This model may be contrasted with Shimer's (2005) search and bargaining model, in which there is no renegotiation on the arrival of an outside offer. In that case expected future competition from other firms affects the current wage, and turnover is not efficient.

⁸An alternative assumption that there is a separate initial bargain with an instantaneous transfer

3 Investment in Human Capital

In this section I determine the choice of human capital, and in particular the mix between general and specific investment, taking as given the expected future opportunities for skilled workers: specifically, the rate λ at which skilled workers encounter alternative potential job offers – which is a measure of the degree of labour market competition.

Let $J(g, s)$ be the expected present value of a match to the worker and firm jointly, when the worker has human capital (g, s) . Given the bargaining assumptions, human capital investment will maximise the joint value of each match. When a worker with human capital g_0 meets a firm, his human capital can be increased to (g, s) at the cost of the additional units, $(g - g_0) + s$. Thus the potential value of a match with idiosyncratic benefit z is:

$$J_0(g_0, z) = \max_{g \geq g_0, s \geq 0} \{J(g, s) - (g - g_0) - s + z\} \quad (1)$$

If (g^*, s^*) is an optimal choice of human capital when $g_0 = 0$, it follows immediately from (1) that the same choice will be optimal subsequently when $g_0 = g^*$. Hence we have:

Lemma 1 *The optimal levels of general and specific human capital are the same for every match. Investment in general capital occurs only when an unskilled worker first meets a firm; re-investment in specific capital occurs in each subsequent match.*

Figure 1 summarises worker flows and corresponding human capital investments. Workers are initially in the unskilled labour market, where job opportunities arrive at rate λ_0 ; they become skilled on obtaining their first job, after which the job-finding rate is λ .

We can now determine $J(g, s)$. While the match continues it produces a flow of output $p_g(g) + p_s(s)$. At rate λ the worker meets a potential alternative employer; he changes jobs if and only if the joint value of the potential match, $J_0(g, z)$, is higher than the current value $J(g, s)$; let x be the reservation z -value for a job change. If turnover occurs, the worker obtains $J(g, s) + \beta(J_0(g, z) - J(g, s))$. At rate δ , the match is destroyed: $J(g, s)$ is lost, but the worker gains $U(g)$, the expected present value of income for an unemployed skilled worker with human capital g . Hence:

$$\begin{aligned} \gamma J(g, s) &= p_g(g) + p_s(s) + \lambda \beta \int_x^{\bar{z}} \{J_0(g, z) - J(g, s)\} dF(z) + \delta(U(g) - J(g, s)) \\ &\text{where } J_0(g, x) = J(g, s) \end{aligned}$$

$U(g)$ is obtained similarly; an unemployed skilled worker with human capital g accepts a

covering training costs would deliver identical results.

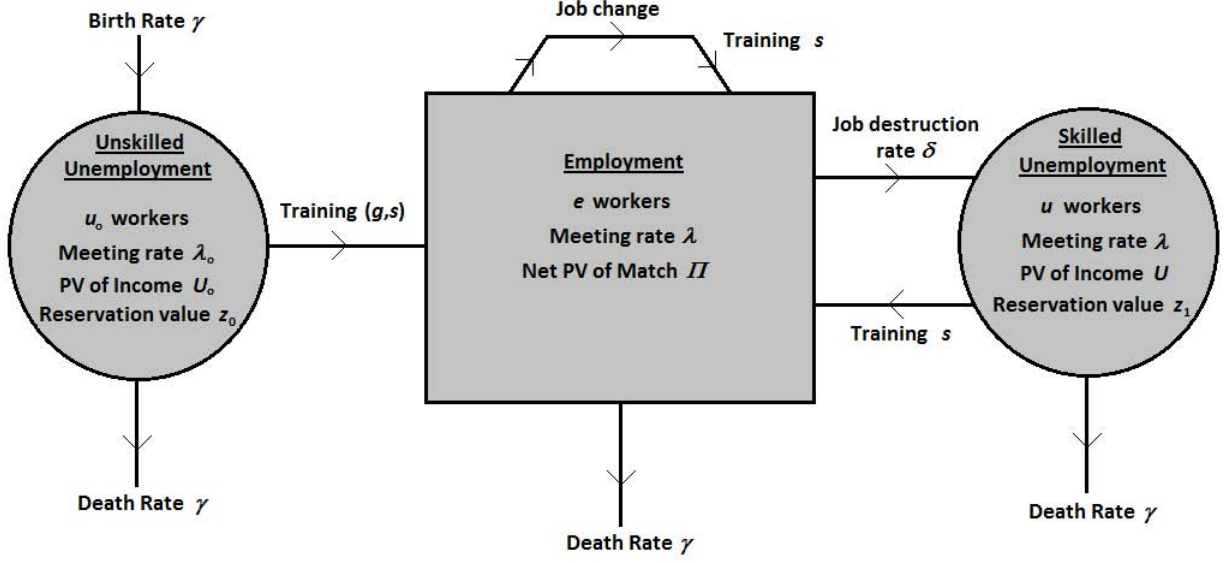


Figure 1: Employment Flows and Training

job if and only if $J_0(g, z) \geq U(g)$. Letting z_1 be the reservation value for acceptance:

$$\gamma U(g) = b + \lambda\beta \int_{z_1}^{\bar{z}} \{J_0(g, z) - U(g)\} dF(z) \quad \text{where } J_0(g, z_1) = U(g)$$

Now define:

$$\Pi(g, s) \equiv J(g, s) - g - s \quad \text{and} \quad \Pi^* \equiv \max_{s \geq 0, g \geq 0} \Pi(s, g)$$

Π is the *net* value of the match, taking into account the cost of the human capital. The initial joint value of a match is then $J_0(g_0, z) = \Pi^* + g_0 + z$. Writing the equations above in terms of Π rather than J , and simplifying the integrals by noting that $\partial J_0 / \partial z = 1$, leads to:

Lemma 2 *The optimal levels of human capital (g^*, s^*) maximise $\Pi(g, s)$ determined by:*

$$(\gamma + \delta)\Pi(g, s) = p_g(g) + p_s(s) - (\gamma + \delta)(g + s) + \lambda\beta \int_x^{\bar{z}} (1 - F(z))dz + \delta U(g) \quad (2)$$

$$\gamma U(g) = b + \lambda\beta \int_{z_1}^{\bar{z}} (1 - F(z))dz \quad (3)$$

subject to:

$$x = \Pi(g, s) - \Pi^* + s \quad (4)$$

$$z_1 = U(g) - \Pi^* - g \quad (5)$$

$$g \geq 0 \quad \text{and} \quad s \geq 0 \quad (6)$$

It is helpful to note that we can solve a simpler problem than the one in Lemma 2:

Lemma 3 (g^*, s^*) is an optimal choice of human capital if and only if it is a solution of the modified problem in which the constraint (4) is replaced by $x = s$.

Lemma 3 is a straightforward application of the result for optimisation of implicit functions given in the Appendix (Lemma 5). It can also be verified directly by deriving the first- and second-order conditions for the two problems. The advantage of working with the modified problem is that the cross-partial derivative of the objective function is identically zero, so the choices of g and s are independent; hence we can determine them separately in sections 3.1 and 3.2, and the comparative statics results in 3.3 are simple to prove.

3.1 Optimal Specific Capital, $s^*(\lambda)$

Writing $\hat{\Pi}(g, s)$ for the objective function in the modified problem in Lemma 3, the first- and second-order conditions for optimal specific capital s^* are:

$$(\gamma + \delta) \frac{\partial \hat{\Pi}}{\partial s} = p'_s(s) - (\gamma + \delta + \lambda\beta(1 - F(s))) \begin{cases} < 0 & s = 0 \\ = 0 & s \geq 0 \end{cases} \quad (7)$$

$$(\gamma + \delta) \frac{\partial^2 \hat{\Pi}}{\partial s^2} = p''_s(s) + \lambda\beta f(s) < 0 \quad (8)$$

There is no reason to assume a unique solution to the first order condition, or that there is necessarily an interior optimum. An assumption that p_s is concave would not be sufficient to guarantee a concave objective function. With higher levels of specific capital the worker is less likely to leave, and this increases the marginal return to specific capital. This increasing returns effect is a generic property of specific training. Even if productivity $p_s(s)$ is strongly concave, the increasing returns effect becomes important when λ is high. I will assume only that $p'_s(\bar{z}) < \gamma + \delta$, which guarantees an optimal value for s below \bar{z} , implying that specific capital never eliminates turnover.

The choice of specific human capital s depends on the job finding rate λ . Let $s^*(\lambda)$ be the optimal investment level. If for particular values of λ the global optimum is not unique I assume without loss of generality that the lowest optimum is chosen – so s^* is a function, continuous and differentiable almost everywhere.

3.2 Optimal General Capital, $g^*(\lambda)$

The first and second-order conditions for optimal general human capital g^* are:

$$(\gamma + \delta) \frac{\partial \hat{\Pi}}{\partial g} = p'_g(g) - \gamma - \frac{\delta\gamma}{\gamma + \lambda\beta(1 - F(z_1))} \begin{cases} < 0 & g = 0 \\ = 0 & g \geq 0 \end{cases} \quad (9)$$

$$(\gamma + \delta) \frac{\partial^2 \hat{\Pi}}{\partial g^2} = p_g''(g) + \frac{\gamma^2 \delta \lambda \beta f(z_1)}{(\gamma + \lambda \beta (1 - F(z_1)))^3} < 0 \quad (10)$$

Note that z_1 is determined by (3) and (5), which have a unique solution for any g . To ensure that an optimum g^* exists, it is sufficient to assume that $\lim_{g \rightarrow \infty} p_g'(g) < \gamma + \delta$. But again the solution to the first-order condition may not be unique. Non-uniqueness arises because there is another increasing returns effect: a worker with higher g has a lower reservation value when unemployed, so spends less time in unemployment, and this raises the return to additional units of human capital. In contrast to the specific capital case, this effect *diminishes* as λ increases (that is, as the labour market becomes more competitive). As before, I allow for non-uniqueness of the global optimum by defining $g^*(\lambda)$ as the lowest optimal investment level.

3.3 Comparative Statics: The Effect of the Job-Finding Rate

The first order conditions suggest that future employment opportunities for skilled workers, represented by the skilled job-finding rate λ , encourage investment in general capital but discourage specific capital. The effect on general capital occurs because a higher λ means that the worker expects to find a skilled job easily if he loses the present one (it is zero if $\delta = 0$). In contrast, a higher λ reduces the expected duration of the current contract and hence the return to specific capital. To confirm that these are global effects (that is, to allow for non-unique solutions to the first-order conditions) we can apply standard monotone comparative statics results (see, for example, Topkis, 1998):

Proposition 1 *Optimal investments in general and specific human capital, $g^*(\lambda)$ and $s^*(\lambda)$, are increasing and decreasing, respectively, in the job finding rate λ .*

PROOF: Since $\frac{\partial^2 \hat{\Pi}}{\partial s \partial g} \equiv 0$ we can treat the two variables separately. Optimal specific capital $s^* \in [0, \bar{z}]$; the objective function $\hat{\Pi}$ has strictly decreasing differences in s and λ since from (7) $\frac{\partial^2 \hat{\Pi}}{\partial s \partial \lambda} < 0 \forall s \in [0, \bar{z}]$ and $\lambda > 0$. Hence if $\lambda_2 > \lambda_1$, $s^*(\lambda_2) \leq s^*(\lambda_1)$. Since $\hat{\Pi}$ also has strictly increasing differences in g and λ (see Appendix), an identical argument implies that g^* is increasing. ■

With s^* and g^* optimally chosen, the net value of a match $\Pi(s^*, g^*)$ increases with the job-finding rate. To see this, put $s = s^*$, $g = g^*$ and $\Pi = \Pi^*$ into equations (2) to (5), and rearrange, to obtain:

$$\int_{z_1}^{s^*} (\gamma + \delta + \lambda \beta (1 - F(z))) dz = p_g(g^*) + p_s(s^*) - b \quad (11)$$

$$\gamma U = b + \lambda \beta \int_{z_1}^{\bar{z}} (1 - F(z)) dz \quad (12)$$

$$\Pi = U - z_1 - g^* \quad (13)$$

These equations determine the reservation value of unemployed workers z_1^* , their expected income U^* , and the expected value of a match Π^* as continuous functions of s^* , g^* , λ . The derivatives with respect to each argument are given in the appendix. In summary⁹:

$$z_1 = z_1^*(s^*, g^*, \lambda) < 0 \quad U = U^*(s^*, g^*, \lambda) \quad \Pi = \Pi^*(s^*, g^*, \lambda) \quad (14)$$

Thus when general human capital investment is high, unemployed skilled workers have high expected income; and since the return to matching is high, they will accept matches with low idiosyncratic value. At the optimal choices, changes in human capital have no first-order effect on the net value Π of a match except at corner solutions; specific capital has no first-order effect on the value of skilled unemployment either, since it is freely chosen after matching. A high job-finding rate improves the value of unemployment, and of future matches, so raises the unemployed worker's reservation value.

Although g^* and s^* depend on λ according to the monotone comparative statics results established in Proposition 1, and may not be differentiable everywhere, the envelope theorem still applies:

$$\frac{d\Pi^*}{d\lambda} = \frac{\partial\Pi^*}{\partial\lambda} > 0 \quad (15)$$

4 Steady State Equilibrium

Assume that in both the skilled and the unskilled labour markets, the rate at which workers and firms meet is determined by a matching function with standard properties: it is increasing and concave in the numbers of agents searching on both sides of the market, with constant returns to scale. Then the job-finding rate for workers can be written as $m(\theta)$, where market tightness, θ , is the ratio of vacancies to searching workers, and the elasticity of m with respect to θ is between 0 and 1; the vacancy-filling rate is $m(\theta)/\theta$.

4.1 The Market for Unskilled Workers

In the unskilled labour market, let u_0 and v_0 be the numbers of unemployed workers and unskilled vacancies. The job-finding rate is $\lambda_0 = m(\theta_0)$ where $\theta_0 = v_0/u_0$.

The expected present value of income for an unemployed worker, U_0 , can be determined as for the skilled labour market. When an unskilled worker meets a firm, the potential joint value of the match is $J_0(0, z)$, where z is the idiosyncratic benefit. They will match if and

⁹Brackets indicate effects that are zero at interior solutions.

only if $z \geq z_0$, where the reservation value z_0 satisfies $J_0(0, z_0) = U_0$; that is, $z_0 = U_0 - \Pi^*$, where Π^* is the expected value of a match with optimal investment. Combining this with a free-entry condition for firms (19) and a steady-state condition (20) we have:

Lemma 4 *A steady-state equilibrium $(\lambda_0, U_0, z_0, \theta_0, u_0)$ in the unskilled labour market satisfies:*

$$\lambda_0 = m(\theta_0) \quad (16)$$

$$\gamma U_0 = b + \lambda_0 \beta \int_{z_0}^{\bar{z}} (1 - F(z)) dz \quad (17)$$

$$z_0 = U_0 - \Pi^* \quad (18)$$

$$\theta_0 c = \lambda_0 (1 - \beta) \int_{z_0}^{\bar{z}} (1 - F(z)) dz \quad (19)$$

$$u_0 = \frac{\gamma}{\gamma + \lambda_0 (1 - F(z_0))} \quad (20)$$

It is straightforward to obtain comparative statics with respect to the match value Π^* , and hence (from (15)) with respect to the skilled job-finding rate λ :

$$\frac{d\lambda_0}{d\lambda} > 0; \quad \frac{dU_0}{d\lambda} > 0; \quad \frac{dz_0}{d\lambda} < 0; \quad \frac{d\theta_0}{d\lambda} > 0; \quad \text{and} \quad \frac{du_0}{d\lambda} < 0 \quad (21)$$

Thus, when the skilled job-finding rate is high, workers in the unskilled labour market have better future opportunities, with more general and less specific training, so lower their reservation value. Matches in the unskilled labour market are worth more, so firms create more vacancies and workers exit faster from unskilled unemployment.

4.2 The Market for Workers with General Skills

The equilibrium value of λ will be determined by job-creation in the market for skilled workers. First note that if there are u unemployed skilled workers and e employed skilled workers, $u_0 + u + e = 1$. With v skilled vacancies, since all skilled workers search, the skilled job-finding rate is:

$$\lambda = m(\theta) \quad \text{where} \quad \theta = \frac{v}{u + e} \quad (22)$$

We have already established (see (14)) the expected income of unemployed skilled workers, U , their reservation value z_1 , and the net value of a match Π , taking λ as given. In equilibrium, λ will be determined by job creation in the skilled labour market, together with a steady-state condition:

$$(\delta + \gamma)e = \lambda_0 u_0 (1 - F(z_0)) + \lambda u (1 - F(z_1)) \quad (23)$$

That is, the flow of workers into employment from both unskilled and skilled unemployment is equal to the outflow due to death and job destruction. Using (20), this can be rearranged to obtain the unemployment rate for skilled workers:

$$\frac{u}{u+e} = \frac{\delta}{\delta + \gamma + \lambda(1 - F(z_1))} \quad (24)$$

4.2.1 Skilled Vacancy Creation

A firm with a skilled vacancy meets both unemployed and employed workers, who have different reservation values, z_1 and s^* respectively, and creates vacancies until the expected return is equal to the cost:

$$c = \frac{\lambda}{\theta}(1 - \beta)Z \quad (25)$$

$$\begin{aligned} \text{where } Z &= \frac{u}{u+e} \int_{z_1}^{\bar{z}} (1 - F(z))dz + \frac{e}{u+e} \int_{s^*}^{\bar{z}} (1 - F(z))dz \\ &= \frac{\delta}{\delta + \gamma + \lambda(1 - F(z_1))} \int_{z_1}^{s^*} (1 - F(z))dz + \int_{s^*}^{\bar{z}} (1 - F(z))dz \\ \text{and } z_1 &= z_1^*(s^*, g^*, \lambda) \end{aligned}$$

So the firm's payoff depends on the human capital of the workers in the skilled labour market, (s^*, g^*) , and the matching rate λ , and we can write the vacancy-creation condition as:

$$c = r(s^*, g^*, \lambda) \quad (26)$$

The return to vacancy creation, r , increases with g^* , because skilled workers are more productive, and decreases with s^* , because employed workers with specific capital are less likely to accept job offers. And r decreases with λ : first because the meeting rate for firms, λ/θ , is low when the meeting rate for workers is high; and secondly Z is decreasing because when λ is high the firm has a higher probability of meeting an employed worker rather than an unemployed one, and employed workers have higher reservation values. These effects are verified formally in the appendix.

Thus the vacancy-creation condition (26) determines the job-finding rate λ , given human capital: $\lambda = \lambda^*(s^*, g^*)$. The interpretation of this relationship is that when g is high and s is low, firms have greater incentives to enter the skilled labour market, so market tightness, and hence the matching rate for workers, are both high. When the workers have high general human capital, they are highly productive, without any need to invest further in human capital. On the other hand, when employed workers have high specific capital, other firms are less likely to be able to recruit them.

4.3 Equilibria

Bringing together the results of sections 3.3 and 4.2, incentives to invest in human capital depend on agents' expectation of the job-finding rate λ for skilled workers: $g = g^*(\lambda^+)$ and $s = s^*(\bar{\lambda})$; while incentives for firms to enter the market for skilled workers, which determine the job-finding rate, depend on the investments that have been made: $\lambda = \lambda^*(s^*, g^*)$. Thus investments in human capital are strategic complements: through the job-finding rate, investments in general (specific) capital increase the return to general (specific) capital for other workers. Substituting for human capital in the vacancy-creation condition (26) determines λ in equilibrium:

$$c = r^*(\lambda) \quad \text{where} \quad r^*(\lambda) \equiv r(s^*(\bar{\lambda}), g^*(\lambda^+), \bar{\lambda}) \quad (27)$$

and the equilibrium is stable if r^* is decreasing in λ at the solution.

Note that as turnover λ increases there are two opposing effects on the firm's return r . The *competition effect* is the standard effect in search models: if workers are matching quickly with other firms, the return to vacancy creation is low. But here we also have an opposing *human capital effect*: higher λ induces more investment in general training and less in specific training, raising the return to vacancy creation.

Proposition 2 *There is at least one stable equilibrium, provided that $\lim_{\theta \rightarrow 0} m'(\theta)$ is sufficiently large. If r^* is downward-sloping for all λ , the equilibrium is unique. Then a fall in costs c , or equivalently a fall in exogenous frictions, leads to an equilibrium with higher turnover.*

PROOF: r^* is a function of λ defined for $\lambda \in [0, \infty)$. First, if the marginal matching rate $m'(\theta)$ is sufficiently large as θ tends to zero, the limit of r^* at $\lambda = 0$ is greater than c (for a Cobb-Douglas matching function it is infinite). Also r^* is decreasing in λ for λ sufficiently small. Secondly, $r^* \rightarrow 0$ as $\lambda \rightarrow \infty$, so r^* is eventually decreasing in λ . For moderate values of λ , r^* may be increasing or decreasing. r^* is not necessarily continuous in λ since investments in human capital can jump. However, at any such points the jump will increase r^* . So there must be at least one intersection between a horizontal line at c and a downward-sloping part of the function r^* . The intersection is unique if r^* slopes down throughout the range, and occurs at lower λ as c increases. ■

From the proof of Proposition 2 it follows that if $r^*(\lambda)$ is increasing over any part of the range – that is, if the human capital effect dominates the competition effect – there will be multiple equilibria for some values of c . Section 4.4 demonstrates that this is indeed possible. But even where $r^*(\lambda)$ is decreasing, the decrease is slowed by the human capital

effects of higher turnover. As illustrated in Figure 2, this means that a small change in frictions, from c_1 to c_2 , may have a large effect on the equilibrium λ , and hence on the character of the equilibrium – which is very different in the high and low turnover cases.

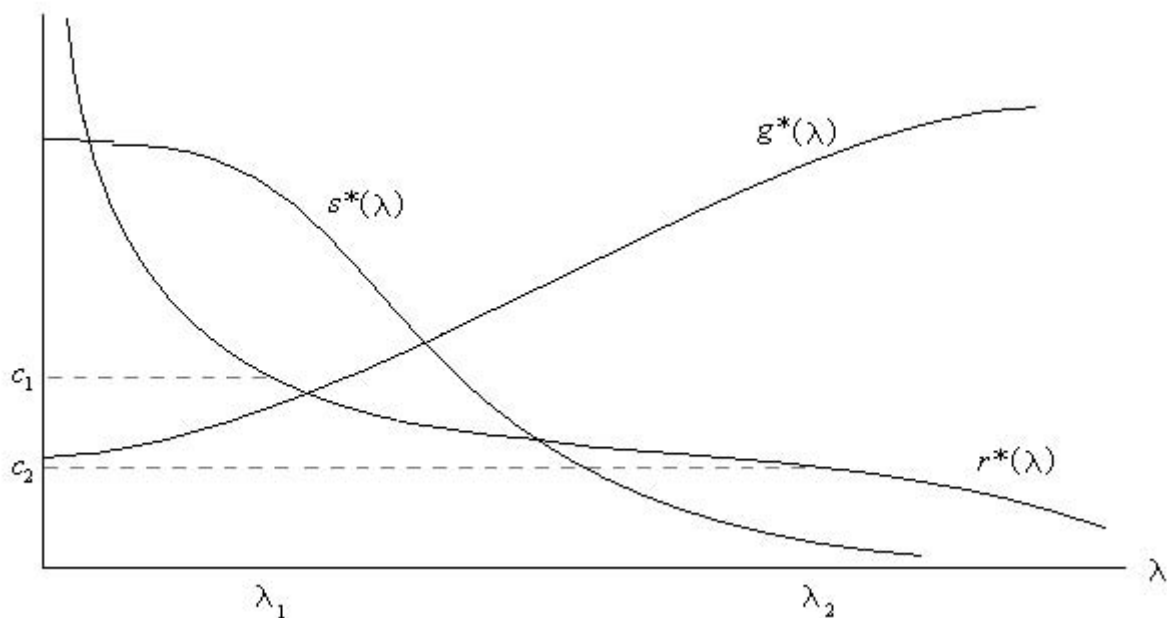


Figure 2: Equilibrium

4.3.1 A high-turnover equilibrium

At an equilibrium with low exogenous frictions and correspondingly high turnover, there will be little investment in specific capital. For λ sufficiently large s^* will be zero. But general human capital investment will be high: it is valuable in all matches, and if any match is destroyed the worker will exit rapidly from unemployment. In addition the worker gains the benefits from finding good matches. Many firms enter the skilled labour market, since they can recruit high productivity workers easily from other firms, so skilled unemployment is low. So too is unemployment for unskilled workers, since workers anticipating high future benefits from investing in general training will have low reservation wages; and although expected tenure is low firms expect to appropriate some of these benefits during the initial period of employment.

Conditions favourable to this type of equilibrium would be a productive general training technology, or substantial match heterogeneity. However, as discussed further in section 5 on wages, it relies on the willingness of workers to bear the costs of training through low wages in the early part of their careers, in the belief that future wages and

labour market opportunities for skilled workers will be high. The features of this equilibrium are consistent with the evidence described earlier for some sectors in Germany, where regulation of apprenticeships arguably helps to align expectations. A stable environment may be important to sustain worker investment: the robustness of the German system to changes in production techniques has been questioned (Culpepper, 1999). High-turnover general training equilibria may be more likely in sectors where skills change little, so that trainees can have confidence in their future opportunities. An example is hairdressing in the UK, where mobility is high but there are well-established programmes combining on- and off-the-job training lasting two or more years, during which time the wage is less than half that of a qualified hairdresser (Druker, Stanworth and White, 2003).

4.3.2 A low-turnover equilibrium

On the other hand when frictions are high, expected tenure and hence specific capital investment will also be high *ceteris paribus* (recall the increasing returns effect) particularly if the exogenous job destruction rate is low. In this case it may still be worthwhile to invest in general capital, which is valuable throughout the long expected tenure. The return will be lower, because if the match is destroyed it will take longer to find a new match, but with a low job destruction rate the loss may be relatively small.

This is the “Japanese” equilibrium, in which we see workers with both specific and general skills, but the direct benefit of turnover, improved matching, is lost. Unemployment may be high for unskilled workers. This type of equilibrium can deliver high welfare provided that match heterogeneity and the job destruction rate are low while the productivity gains from either general or specific training are high. An increase in job destruction, as happened in Japan during the 1990s, would reduce incentives for both specific and general investment.

4.4 Multiple Equilibria

As noted above, multiple equilibria arise for some values of c when over some part of the range the human capital effect of λ , which increases the firm’s return r^* , dominates the competition effect – the fall in expected return due to the vacancies created by other firms. As usual there will be an odd number of equilibria; stable equilibria where r^* is decreasing and intermediate unstable ones where it is increasing.

Where multiple equilibria exist, those with higher λ have higher general capital, lower specific capital, and higher labour turnover. Moreover, equilibria are welfare-ranked according to λ . Social welfare is aggregate steady-state income – which, with no discounting,

is equal to the permanent income of a worker at birth, γU_0 . And with optimally-chosen human capital, U_0 is increasing in λ (see (21)). If multiple equilibria exist for given training technologies and match heterogeneity, the benefits of being at a high turnover equilibrium (from matching, lower unemployment, and investment in general skills) always outweigh the cost (lower specific capital).

Whether multiple equilibria can actually occur depends on the functional forms of $p_s(s)$, $p_g(g)$, and $F(z)$. Below I analyse a simple case where p_s and p_g are step functions. However, it can be verified that multiple equilibria are possible without any such non-convexity: the appendix specifies a particular case.

4.4.1 An Example

Suppose that:
$$p_s(s) = \begin{cases} 0 & s < 1 \\ p & s \geq 1 \end{cases} \quad \text{and} \quad p_g(g) - b = \begin{cases} a & g < 1 \\ a + q & g \geq 1 \end{cases}$$

where p , q and a are positive constants. Then there are four possible pure strategy equilibria, with g and s taking the values 0 or 1. Suppose also that the distribution of idiosyncratic values z has mean μ , and $\bar{z} > 1$, and $\kappa = \int_0^1 (1 - F(z)) dz < \mu$; κ represents the potential loss of idiosyncratic benefits due to reduced turnover when s takes the value one rather than zero.

The parameter space consists of the productivity of human capital, (p, q) , the cost parameters γ , δ and c (the cost of maintaining a vacancy), and the characteristics of the distribution of idiosyncratic match value, μ and κ . Using Lemmas 2 and 3, and equations (11) and (25), we can solve for s^* , g^* , z_1 and r in terms of λ . The exact expressions are given in the Appendix, but there are threshold values $\hat{\lambda}_q$, where general training begins, and $\hat{\lambda}_p$ where specific training ceases. Equilibrium values of λ satisfy the free-entry condition $r = c$. Three cases of multiple equilibria are described below.

Case (i): $p > \gamma + \delta$, $q \in (\gamma, \gamma + \delta]$ and $\hat{\lambda}_p > \hat{\lambda}_q$

When λ is low, there is investment in specific capital only. r falls as λ increases, due to increasing competition and also the falling return to specific capital as turnover rises. At the threshold $\hat{\lambda}_q$ it becomes worthwhile to invest in general capital, and the return jumps upward. Then it falls again with λ until at the threshold $\hat{\lambda}_p$ it is no longer worth investing in specific capital; the return jumps again because employed workers are more likely to change jobs and gain k in idiosyncratic benefit. When λ is high the return falls due to the effect of competition, although this is partially offset by the rising return to general capital as unemployment falls.

Figure 3 shows a value of c for which there are three stable equilibria, welfare-ranked according to λ . Whether all three exist in any particular case depends not only on the

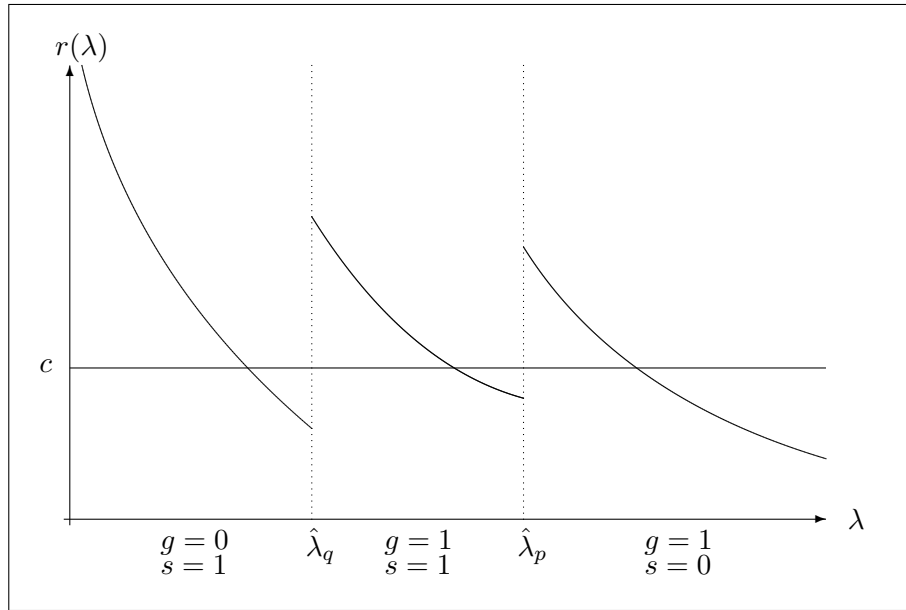


Figure 3: Multiple Equilibria in Case (i)

cost c , but also on the gains q and κ from general training and turnover respectively, which determine the size of the jumps. If $\hat{\lambda}_q > \hat{\lambda}_p$ the picture is similar, but s and g are both zero in the intermediate region.

Case (ii): $p < \gamma + \delta$ and $q \in (\gamma, \gamma + \delta]$

In this case there is no investment in specific capital. However, they may still be multiple equilibria as shown in Figure 4. At the inefficient equilibrium, turnover is too low for investment in general capital to be worthwhile, and firms create few vacancies for “experienced” workers (who are identical to unskilled workers) because they are not very productive.

Case (iii): $p > \gamma + \delta$ and $q > \gamma + \delta$

Here the return to general capital is large enough that there is always general training. For some values of c there are two equilibria: a low turnover equilibrium with $\lambda < \hat{\lambda}_p$ in which specific training occurs, and an efficient high-turnover equilibrium without specific capital.

5 Wages, Cost-Sharing, and Externalities

Wages have played little part in the analysis because I have assumed throughout that they are determined under full information, so that turnover is fully efficient. It is of course possible to solve for the implied wage profiles, as in Cahuc et al (2006), but without doing so we can describe qualitatively how the wage profile is related to turnover and human

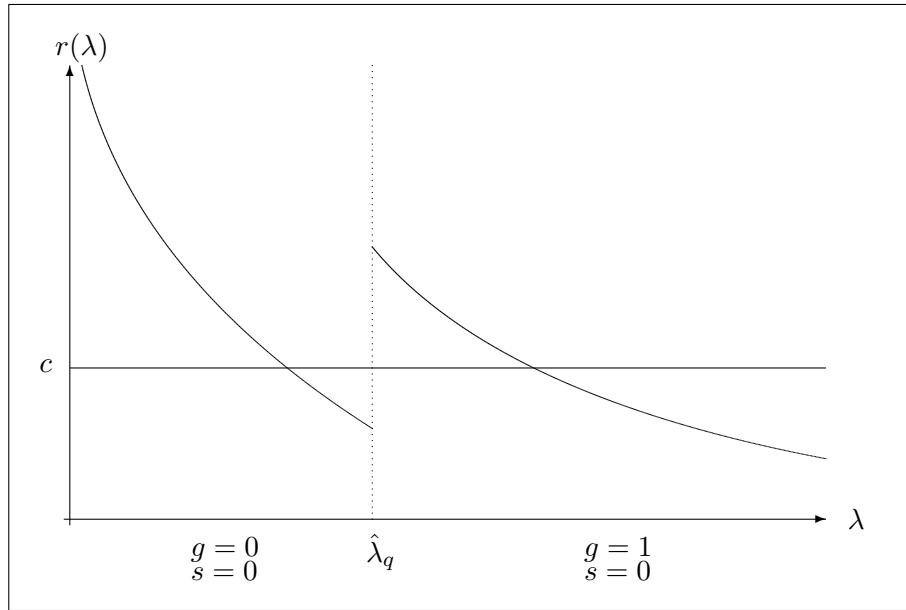


Figure 4: Multiple Equilibria in Case (ii)

capital investment.

The costs of both specific and general capital are reflected in the wage that prevails between the time of the investment, and the arrival of the first binding alternative offer. In a high-turnover equilibrium with mainly general human capital, this initial wage will be very low - because it is only during this short initial period that the firm can expect to extract its share of the surplus. When an alternative offer arrives, the wage will be renegotiated if the alternative match has higher value the value of the current contract to the worker - and outside offers will tend to be high for a generally-skilled worker. Once the worker has obtained a binding outside offer, the firm can no longer capture any of the returns to general training. In the competitive limit, as frictions tend to zero, all general training costs are borne by the worker in an initial instantaneous transfer. On the other hand, in a low turnover equilibrium there will be a long initial period before the worker obtains a binding outside offer. Even if he has general skills as well, a larger part of the return to them will be obtained within the original match, so will be shared by the firm. In this case the wage profile will be relatively flat. But there is a distinction between specific and general training: for specific training the firm can continue to share in the returns throughout the duration of the match, even after a binding outside offer has arrived, although the firm's share will diminish as better outside offers are received.

In summary, the main determinant of the slope of the wage profile, and hence the extent to which training costs are shared, is the expected match duration. But *ceteris paribus* it will be somewhat steeper for general than for specific training. Note also that

when a worker, whether skilled or unskilled, is recruited from unemployment, the initial wage will be low irrespective of training costs, because the worker’s outside option is unemployment and the firm is able to extract some of the initial match rent.

If workers were credit constrained and could not accept low initial wages, or if as in Dustmann and Schönberg (2012) firms were unable to commit to training provision, an equilibrium with high general training and turnover would not be feasible. With less investment in general training, incentives to create skilled jobs would be lower, reducing turnover and raising the returns to specific investment. We should expect to see an equilibrium with longer tenure and flatter wage profiles, even if some of the training was general.

Finally, an obvious question that can be addressed using this model is the extent to which “poaching” of skilled workers causes underinvestment in general training. It might appear that with on-the-job search and firms freely able to create skilled jobs in anticipation of recruiting workers from other firms, this would be a serious problem. But with no market imperfections other than the frictions, this is not the case. When job-to-job turnover is high, workers capture the return to general training and bear most of the costs. A firm recruiting an employed worker must reward him fully for his general human capital, so there is no “poaching externality” associated with job-to-job moves of the kind that would arise if wage determination were less competitive (for example if outside offers were not matched, as in Shimer (2005)).

There is still an externality associated with general training, because firms recruiting unemployed skilled workers can extract rent. So frictions *do* cause underinvestment in general training. Despite this, skilled job creation has essentially positive effects in this model; it leads to more job creation in the unskilled labour market and more general training.

6 Conclusions

The model presented in this paper shows how training, turnover and tenure are jointly determined when there are frictions in the labour market, and the arrival rate of job opportunities determines the supply and demand for training. I have allowed for three types of human capital investment: specific, general and job matching. Although these can all be determined independently – there are no technological links between them – the model demonstrates that they are closely related strategically. So, if agents invest in specific capital then the returns from job matching and general training are reduced; and when general training and the benefits from matching are high, there is little or no

benefit from specific training.

Strategic complementarities thus give rise to labour markets with particular combinations of characteristics, consistent with the stylised examples of Japan and Germany. In the US and the UK, which have often been compared unfavourably with both of these polar cases, it may be more helpful to characterise training systems by sector. But to the extent that the evidence paints an economy-wide picture, it is consistent with an intermediate position, where turnover of skilled workers is low enough to allow some investment in both general and specific training, financed mainly by firms, but high enough to generate some gains from job matching. In this situation the model suggests that returns to both specific and general human capital may be relatively low, even if the combined training investment is high. It is possible in principle that a better, high-turnover, equilibrium exists, or that small changes could increase equilibrium turnover and welfare substantially.

As noted in the introduction, it is usually assumed that a high turnover environment is bad for investment in skills. It is certainly true that an exogenous reduction in turnover would increase investment in specific training and also the willingness of firms to bear costs of general training. But in a general equilibrium setting, turnover and training are jointly determined. In the model in this paper, an increase in frictions would reduce turnover, and could increase total training investment, but it would be unambiguously bad for equilibrium welfare. This is not to deny that lowering turnover by increasing frictions could be a second-best response to the presence of credit constraints (as in Stevens, 2001).

In practice, high-turnover equilibria may be fragile, and require institutional support, since they rely critically on the willingness of workers to accept lower wages during training, in anticipation of better future opportunities. Turnover is determined by workers' collective decisions, so it not enough for an individual worker to be able and willing to finance his own training, if others do not do so; he needs to be able to rely on the existence of a skilled labour market where his investment will be rewarded.

Appendix

A.1 (Section 3)

Lemma 5 (*Optimisation of Implicit Functions.*) *If the function $\pi(x)$ is defined implicitly for $x \in X \subseteq \mathfrak{R}^n$ by $\pi(x) = f(x, \pi(x))$, where $f_\pi < 1$, then:*

$$\begin{aligned} x^* &= \arg \max_{x \in X} \pi(x) & \iff & x^* = \arg \max_{x \in X} f(x, \pi^*) \\ \pi^* &= \pi(x^*) & & \pi^* = f(x^*, \pi^*) \end{aligned}$$

PROOF: Assuming a single variable x , from the definition of $\pi(x)$ the derivatives of f and π always have the same sign: $\pi'(x)(1 - f_2(x, \pi)) = f_1(x, \pi)$. So the first-order conditions (for both interior optima and corner solutions) are the same for both problems. For the second order conditions: $\pi''(x)(1 - f_2) = f_{11} + f_{21}\pi'(x) + f_{22}(\pi'(x))^2$. At an interior optimum, $\pi''(x) = f_{11}$, so the second order conditions are the same for both problems. (The proof extends easily to functions of several variables.) \blacksquare

A.2 (Section 3.3)

A.2.1 $\hat{\Pi}(s, g)$ has increasing differences in g and λ

From (9), (3) and (5):

$$\begin{aligned}
(\gamma + \delta) \frac{\partial \hat{\Pi}}{\partial g} &= p'_g(g) - \gamma - \frac{\delta \gamma}{\gamma + \lambda \beta (1 - F(z_1))} \quad \text{where} \quad \gamma(z_1 + \Pi^* + g) = b + \lambda \beta \int_{z_1}^{\bar{z}} (1 - F(z)) dz \\
\Rightarrow \frac{\partial^2 \hat{\Pi}}{\partial \lambda \partial g} &\stackrel{\text{sgn}}{=} \frac{\partial}{\partial \lambda} (\lambda (1 - F(z_1))) = 1 - F(z_1) - \lambda f(z_1) \frac{\partial z_1}{\partial \lambda} \\
&= 1 - F(z_1) - \frac{\lambda \beta f(z_1) \int_{z_1}^{\bar{z}} (1 - F(z)) dz}{\gamma + \lambda \beta (1 - F(z_1))} > 1 - F(z_1) - \frac{f(z_1) \int_{z_1}^{\bar{z}} (1 - F(z)) dz}{1 - F(z_1)}
\end{aligned}$$

which is positive since F is log-concave and hence so is $\int_{z_1}^{\bar{z}} (1 - F(z)) dz$.

A.2.2 Derivatives of z_1^*, U^*, Π^*

$$\begin{aligned}
z_1 = z_1^*(s^*, g^*, \lambda) \text{ where} \quad & \frac{\partial z_1^*}{\partial s^*} = \frac{\gamma + \delta + \lambda \beta (1 - F(s^*)) - p'_s(s^*)}{(\gamma + \delta + \lambda \beta (1 - F(z_1)))} \geq 0 \\
& \frac{\partial z_1^*}{\partial g^*} = \frac{-p'_g(g^*)}{(\gamma + \delta + \lambda \beta (1 - F(z_1)))} < 0 \\
& \frac{\partial z_1^*}{\partial \lambda} = \frac{\beta \int_{z_1}^{s^*} (1 - F(z)) dz}{(\gamma + \delta + \lambda \beta (1 - F(z_1)))} > 0 \\
U = U^*(s^*, g^*, \lambda) \text{ where} \quad & \gamma \frac{\partial U^*}{\partial s^*} = -\lambda \beta (1 - F(z_1)) \frac{\partial z_1^*}{\partial s^*} \leq 0 \\
& \gamma \frac{\partial U^*}{\partial g^*} = -\lambda \beta (1 - F(z_1)) \frac{\partial z_1^*}{\partial g^*} > 0 \\
& \gamma \frac{\partial U^*}{\partial \lambda} = \beta \int_{s^*}^{\bar{z}} (1 - F(z)) dz + \frac{(\gamma + \delta) \beta \int_{z_1}^{s^*} (1 - F(z)) dz}{(\gamma + \delta + \lambda \beta (1 - F(z_1)))} > 0 \\
\Pi = \Pi^*(s^*, g^*, \lambda) \text{ where} \quad & \gamma \frac{\partial \Pi^*}{\partial s^*} = -(\gamma + \lambda \beta (1 - F(z_1))) \frac{\partial z_1^*}{\partial s^*} \leq 0
\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi^*}{\partial g^*} &= \frac{p'_g(g^*)(\gamma + \lambda\beta(1 - F(z_1)))}{\gamma(\gamma + \delta + \lambda\beta(1 - F(z_1)))} - 1 \leq 0 \\ \gamma \frac{\partial \Pi^*}{\partial \lambda} &= \beta \int_{s^*}^{\bar{z}} (1 - F(z)) dz + \frac{\delta \beta \int_{z_1}^{s^*} (1 - F(z)) dz}{(\gamma + \delta + \lambda\beta(1 - F(z_1)))} > 0\end{aligned}$$

Lastly, to show that $z_1 < 0$, let $\Pi_0 = \Pi(0, g^*)$. From equations (2) to (5):

$$\int_{z_1}^{\Pi_0 - \Pi^*} (\gamma + \delta + \lambda\beta(1 - F(z))) dz = p_g(g^*) - b > 0 \quad \Rightarrow \quad z_1 < \Pi_0 - \Pi^* < 0$$

A.3 (Section 4.2.1)

Write $Z = \hat{Z}(z_1, s^*, \lambda) \equiv \frac{\delta}{\delta + \gamma + \lambda(1 - F(z_1))} \int_{z_1}^{s^*} (1 - F(z)) dz + \int_{s^*}^{\bar{z}} (1 - F(z)) dz$

$$\begin{aligned}\text{Then: } \frac{\partial \hat{Z}}{\partial z_1} &= \frac{\delta \lambda f(z_1)}{(\delta + \gamma + \lambda(1 - F(z_1)))^2} \int_{z_1}^{s^*} (1 - F(z)) dz - \frac{\delta(1 - F(z_1))}{\delta + \gamma + \lambda(1 - F(z_1))} \\ &\stackrel{\text{sgn}}{=} \lambda f(z_1) \int_{z_1}^{s^*} (1 - F(z)) dz - (1 - F(z_1))(\delta + \gamma + \lambda(1 - F(z_1))) \\ &< \lambda f(z_1) \int_{z_1}^{\bar{z}} (1 - F(z)) dz - \lambda(1 - F(z_1))^2 \\ &< 0 \quad \text{since } F \text{ is log-concave and hence so is } \int_{z_1}^{\bar{z}} (1 - F(z)) dz \\ \frac{\partial \hat{Z}}{\partial s^*} &= \frac{\delta(1 - F(s^*))}{\delta + \gamma + \lambda(1 - F(z_1))} - (1 - F(s^*)) < 0 \\ \text{and } \frac{\partial \hat{Z}}{\partial \lambda} &= -\frac{\delta(1 - F(z_1))}{(\delta + \gamma + \lambda(1 - F(z_1)))^2} \int_{z_1}^{s^*} (1 - F(z)) dz < 0\end{aligned}$$

Now: $z_1 = {}_1^*(s^*, g^*, \lambda)$, $r(s^*, g^*, \lambda) \equiv \frac{\lambda}{\theta}(1 - \beta)\hat{Z}(z_1^*(s^*, g^*, \lambda), s^*, \lambda)$

Let $\alpha \in (0, 1)$ be the elasticity of $\lambda = m(\theta)$ with respect to θ . The derivatives of r are:

$$\begin{aligned}\frac{\partial r}{\partial s^*} &\stackrel{\text{sgn}}{=} \frac{\partial \hat{Z}}{\partial s^*} + \frac{\partial \hat{Z}}{\partial z_1} \frac{\partial z_1^*}{\partial s^*} < 0 \\ \frac{\partial r}{\partial g^*} &\stackrel{\text{sgn}}{=} \frac{\partial \hat{Z}}{\partial z_1} \frac{\partial z_1^*}{\partial g^*} > 0 \\ \frac{1}{\hat{r}} \frac{\partial r}{\partial \lambda} &= \frac{1}{\hat{Z}} \left\{ \frac{\partial \hat{Z}}{\partial \lambda} + \frac{\partial \hat{Z}}{\partial z_1} \frac{\partial z_1^*}{\partial \lambda} \right\} - \frac{\alpha}{(1 - \alpha)\lambda} < 0\end{aligned}$$

A.4 (Section 4.4)

An example of multiple equilibria when all functions are well-behaved can be constructed by letting $p'_s(s) = \gamma + \delta + k(1 - F(s))^{3/2}$ (where k is a positive constant and $p_s(0) = 0$), and $m = \theta^{1/2}$. Then s^* satisfies $\lambda\beta = k(1 - F(s^*))^{1/2}$, and the second integral in r ,

$\frac{1-\beta}{\lambda} \int_{s^*}^{\bar{z}} (1 - F(z)) dz$, can then be written as a function of λ that does not depend on δ . If F is log-concave, this integral is increasing in λ for all $s > 0$. And if δ is sufficiently small, the increase in this integral dominates the first term, whatever the functional form of p_g , except when λ is very small, until the point where s goes to zero. In this example multiple equilibria are generated by the effects of specific capital only.

A.5 (Section 4.4.1)

$$\begin{aligned}
s^* &= \begin{cases} 1 & \text{if } \lambda < \hat{\lambda}_p \\ 0 & \text{if } \lambda \geq \hat{\lambda}_p \end{cases} \quad \text{where } \hat{\lambda}_p = \frac{p - (\gamma + \delta)}{\beta\kappa} \\
g^* &= \begin{cases} 1 & \text{if } \begin{cases} q > \gamma + \delta, \text{ or} \\ q \in (\gamma, \gamma + \delta] \text{ and } \lambda > \hat{\lambda}_q \end{cases} \\ 0 & \text{if } \begin{cases} q \leq \gamma, \text{ or} \\ q \in (\gamma, \gamma + \delta] \text{ and } \lambda \leq \hat{\lambda}_q \end{cases} \end{cases} \quad \text{where } \hat{\lambda}_q = \frac{\gamma(\gamma + \delta - q)}{\beta(q - \gamma)} \\
z_1 &= -\frac{a + qg^* + s^*(p - (\gamma + \delta + \lambda\beta\kappa))}{\gamma + \delta + \lambda\beta} \\
r &= \frac{\lambda(1 - \beta)}{\theta} \left\{ \mu - \frac{z_1\delta + s^*\kappa(\mu + \lambda)}{\gamma + \delta + \lambda} \right\}
\end{aligned}$$

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